

## NAG C Library Function Document

### nag\_condl\_logistic (g11cac)

#### 1 Purpose

G11CAF returns parameter estimates for the conditional logistic analysis of stratified data, for example, data from case-control studies and survival analyses.

#### 2 Specification

```
void nag_condl_logistic (Nag_OrderType order, Integer n, Integer m, Integer ns,
    const double z[], Integer pdz, const Integer isz[], Integer p,
    const Integer ic[], const Integer isi[], double *dev, double b[], double se[],
    double sc[], double cov[], Integer nca[], Integer nct[], double tol,
    Integer maxit, Integer iprint, const char *outfile, NagError *fail)
```

#### 3 Description

In the analysis of binary data, the logistic model is commonly used. This relates the probability of one of the outcomes, say  $y = 1$ , to  $p$  explanatory variates or covariates by

$$\text{Prob}(y = 1) = \frac{\exp(\alpha + z^T \beta)}{1 + \exp(\alpha + z^T \beta)},$$

where  $\beta$  is a vector of unknown coefficients for the covariates  $z$  and  $\alpha$  is a constant term. If the observations come from different strata or groups,  $\alpha$  would vary from strata to strata. If the observed outcomes are independent then the  $y$ s follow a Bernoulli distribution, i.e., a binomial distribution with sample size one and the model can be fitted as a generalized linear model with binomial errors.

In some situations the number of observations for which  $y = 1$  may not be independent. For example, in epidemiological research, case-control studies are widely used in which one or more observed cases are matched with one or more controls. The matching is based on fixed characteristics such as age and sex, and is designed to eliminate the effect of such characteristics in order to more accurately determine the effect of other variables. Each case-control group can be considered as a stratum. In this type of study the binomial model is not appropriate, except if the strata are large, and a conditional logistic model is used. This considers the probability of the cases having the observed vectors of covariates given the set of vectors of covariates in the strata. In the situation of one case per stratum, the conditional likelihood for  $n_s$  strata can be written as

$$L = \prod_{i=1}^{n_s} \frac{\exp(z_i^T \beta)}{[\sum_{l \in S_i} \exp(z_l^T \beta)]}, \quad (1)$$

where  $S_i$  is the set of observations in the  $i$ th stratum, with associated vectors of covariates  $z_l$ ,  $l \in S_i$ , and  $z_i$  is the vector of covariates of the case in the  $i$ th stratum. In the general case of  $c_i$  cases per strata then the full conditional likelihood is

$$L = \prod_{i=1}^{n_s} \frac{\exp(s_i^T \beta)}{[\sum_{l \in C_i} \exp(s_l^T \beta)]}, \quad (2)$$

where  $s_i$  is the sum of the vectors of covariates for the cases in the  $i$ th stratum and  $s_l$ ,  $l \in C_i$  refer to the sum of vectors of covariates for all distinct sets of  $c_i$  observations drawn from the  $i$ th stratum. The conditional likelihood can be maximized by a Newton-Raphson procedure. The covariances of the parameter estimates can be estimated from the inverse of the matrix of second derivatives of the logarithm of the conditional likelihood, while the first derivatives provide the score function,  $U_j(\beta)$ , for  $j = 1, 2, \dots, p$ , which can be used for testing the significance of parameters.

If the strata are not small,  $C_i$  can be large so to improve the speed of computation, the algorithm in Howard (1972) and described by Krailo and Pike (1984) is used.

A second situation in which the above conditional likelihood arises is in fitting Cox's proportional hazard model (see `nag_surviv_cox_model` (`g12bac`)) in which the strata refer to the risk sets for each failure time and where the failures are cases. When ties are present in the data `nag_surviv_cox_model` (`g12bac`) uses an approximation. For an exact estimate, the data can be expanded using `nag_surviv_risk_sets` (`g12zac`) to create the risk sets/strata and `nag_condl_logistic` (`g11cac`) used.

## 4 References

Cox D R (1972b) Regression models in life tables (with discussion) *J. Roy. Statist. Soc. Ser. B* **34** 187–220

Cox D R and Hinkley D V (1974) *Theoretical Statistics* Chapman and Hall

Howard S (1972) Remark on the paper by Cox, D.R. (1972): Regression methods *J. R. Statist. Soc.* **B 34** and life tables 187–220

Kraile M D and Pike M C (1984) Algorithm AS 196. Conditional multivariate logistic analysis of stratified case-control studies *Appl. Statist.* **33** 95–103

Smith P G, Pike M C, Hill P, Breslow N E and Day N E (1981) Algorithm AS 162. Multivariate conditional logistic analysis of stratum-matched case-control studies *Appl. Statist.* **30** 190–197

## 5 Parameters

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order = Nag\_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.  
*Constraint:* **order = Nag\_RowMajor** or **Nag\_ColMajor**.
- 2: **n** – Integer *Input*  
*On entry:* the number of observations,  $n$ .  
*Constraint:*  $n \geq 2$ .
- 3: **m** – Integer *Input*  
*On entry:* the number of covariates in array **z**.  
*Constraint:*  $m \geq 1$ .
- 4: **ns** – Integer *Input*  
*On entry:* the number of strata,  $n_s$ .  
*Constraint:*  $ns \geq 1$ .
- 5: **z[dim]** – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **z** must be at least  $\max(1, \mathbf{pdz} \times \mathbf{m})$  when **order = Nag\_ColMajor** and at least  $\max(1, \mathbf{pdz} \times \mathbf{n})$  when **order = Nag\_RowMajor**.  
If **order = Nag\_ColMajor**, the  $(i, j)$ th element of the matrix  $Z$  is stored in  $\mathbf{z}[(j-1) \times \mathbf{pdz} + i - 1]$  and if **order = Nag\_RowMajor**, the  $(i, j)$ th element of the matrix  $Z$  is stored in  $\mathbf{z}[(i-1) \times \mathbf{pdz} + j - 1]$ .  
*On entry:* the  $i$ th row must contain the covariates which are associated with the  $i$ th observation.
- 6: **pdz** – Integer *Input*  
*On entry:* the stride separating matrix row or column elements (depending on the value of **order**) in the array **z**.

Constraints:

if **order** = **Nag\_ColMajor**, **pdz**  $\geq$  **n**;  
if **order** = **Nag\_RowMajor**, **pdz**  $\geq$  **m**.

- 7: **isz[m]** – const Integer *Input*  
*On entry:* indicates which subset of covariates are to be included in the model.  
 If **isz[j – 1]**  $\geq$  1, the *j*th covariate is included in the model.  
 If **isz[j – 1]** = 0, the *j*th covariate is excluded from the model and not referenced.  
*Constraint:* **isz[j – 1]**  $\geq$  0 and at least one value must be non-zero.
- 8: **p** – Integer *Input*  
*On entry:* the number of covariates included in the model, *p*, as indicated by **isz**.  
*Constraint:* **p**  $\geq$  1 and **p** = number of non-zero values of **isz**.
- 9: **ic[n]** – const Integer *Input*  
*On entry:* indicates whether the *i*th observation is a case or a control.  
 If **ic[i – 1]** = 0, indicates that the *i*th observation is a case.  
 If **ic[i – 1]** = 1, indicates that the *i*th observation is a control.  
*Constraint:* **ic[i – 1]** = 0 or 1, for *i* = 1, 2, ..., **n**.
- 10: **isi[n]** – const Integer *Input*  
*On entry:* stratum indicators which also allow data points to be excluded from the analysis.  
 If **isi[i – 1]** = *k*, indicates that the *i*th observation is from the *k*th stratum, where *k* = 1, 2, ..., **ns**.  
 If **isi[i – 1]** = 0, indicates that the *i*th observation is to be omitted from the analysis.  
*Constraint:*  $0 \leq \mathbf{isi}[i - 1] \leq \mathbf{ns}$  and more than **p** values of **isi[i – 1]** > 0, for *i* = 1, 2, ..., **n**.
- 11: **dev** – double \* *Output*  
*On exit:* the deviance, that is,  $-2 \times$  (maximized log marginal likelihood).
- 12: **b[p]** – double *Input/Output*  
*On entry:* initial estimates of the covariate coefficient parameters  $\beta$ . **b[j – 1]** must contain the initial estimate of the coefficient of the covariate in **z** corresponding to the *j*th non-zero value of **isz**.  
*Suggested values:* in many cases an initial value of zero for **b[j – 1]** may be used. For another suggestion see Section 8.  
*On exit:* **b[j – 1]** contains the estimate  $\hat{\beta}_i$  of the coefficient of the covariate stored in the *i*th column of **z** where *i* is the *j*th non-zero value in the array **isz**.
- 13: **se[p]** – double *Output*  
*On exit:* **se[j – 1]** is the asymptotic standard error of the estimate contained in **b[j – 1]** and score function in **sc[j – 1]**, for *j* = 1, 2, ..., **p**.
- 14: **sc[p]** – double *Output*  
*On exit:* **sc[j]** is the value of the score function  $U_j(\beta)$  for the estimate contained in **b[j – 1]**.
- 15: **cov[dim]** – double *Output*  
**Note:** the dimension, *dim*, of the array **cov** must be at least  $\mathbf{p} \times (\mathbf{p} + 1)/2$ .

*On exit:* the variance-covariance matrix of the parameter estimates in **b** stored in packed form by column, i.e., the covariance between the parameter estimates given in **b**[ $i - 1$ ] and **b**[ $j - 1$ ],  $j \geq i$ , is given in **cov**[ $j(j - 1)/2 + i$ ].

- 16: **nca**[**ns**] – Integer *Output*  
*On exit:* **nca**[ $i - 1$ ] contains the number of cases in the  $i$ th stratum, for  $i = 1, 2, \dots, \mathbf{ns}$ .
- 17: **nct**[**ns**] – Integer *Output*  
*On exit:* **nct**[ $i - 1$ ] contains the number of controls in the  $i$ th stratum, for  $i = 1, 2, \dots, \mathbf{ns}$ .
- 18: **tol** – double *Input*  
*On entry:* indicates the accuracy required for the estimation. Convergence is assumed when the decrease in deviance is less than **tol**  $\times$  (1.0 + *CurrentDeviance*). This corresponds approximately to an absolute accuracy if the deviance is small and a relative accuracy if the deviance is large.  
*Constraint:* **tol**  $\geq 10 \times$  *machine precision*.
- 19: **maxit** – Integer *Input*  
*On entry:* the maximum number of iterations required for computing the estimates. If **maxit** is set to 0 then the standard errors, the score functions and the variance-covariance matrix are computed for the input value of  $\beta$  in **b** but  $\beta$  is not updated.  
*Constraint:* **maxit**  $\geq 0$ .
- 20: **iprint** – Integer *Input*  
*On entry:* indicates if the printing of information on the iterations is required. If **iprint**  $\leq 0$ , there is no printing. If **iprint**  $\geq 1$ , the deviance and the current estimates are printed every **iprint** iterations.  
*Suggested value:* **iprint** = 0
- 21: **outfile** – char \* *Input*  
*On entry:* the name of a file to which diagnostic output will be directed. If **outfile** is **NULL** the diagnostic output will be directed to standard output.
- 22: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 2$ .

On entry, **pdz** =  $\langle value \rangle$ .

Constraint: **pdz**  $> 0$ .

On entry, **ic** must be 0 or 1: **ic**[ $\langle value \rangle$ ] =  $\langle value \rangle$ .

On entry, **p** =  $\langle value \rangle$ .

Constraint: **p**  $\geq 1$ .

On entry, **maxit** =  $\langle value \rangle$ .

Constraint: **maxit**  $\geq 0$ .

On entry, element  $\langle value \rangle$  of **isz**  $< 0$ .

On entry, **pdz**  $< \mathbf{n}$ : **pdz** =  $\langle value \rangle$ .

On entry, **ns** =  $\langle value \rangle$ .

Constraint: **ns**  $\geq$  1.

On entry, **m** =  $\langle value \rangle$ .

Constraint: **m**  $\geq$  1.

#### NE\_INT\_2

On entry, **pdz** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .

Constraint: **pdz**  $\geq$  **n**.

On entry, **pdz** =  $\langle value \rangle$ , **m** =  $\langle value \rangle$ .

Constraint: **pdz**  $\geq$  **m**.

On entry, **isi**[ $\langle value \rangle$ ] =  $\langle value \rangle$ ,  $value < 0$  or  $> ns$ : **ns** =  $\langle value \rangle$ .

#### NE\_CONVERGENCE

Convergence not achieved in  $\langle value \rangle$  iterations.

#### NE\_INT\_ARRAY\_ELEM\_CONS

On entry, there are not **p** values of **isz**  $>$  0.

#### NE\_OBSERVATIONS

On entry, too few observations included in model.

#### NE\_OVERFLOW

Overflow in calculations.

#### NE\_REAL

On entry, **tol**  $<$   $10 \times machine\ precision$ : **tol** =  $\langle value \rangle$ .

#### NE\_SINGULAR

The matrix of second partial derivatives is singular; computation abandoned.

#### NE\_ALLOC\_FAIL

Memory allocation failed.

#### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

#### NE\_NOT\_WRITE\_FILE

Cannot open file  $\langle value \rangle$  for writing.

#### NE\_NOT\_CLOSE\_FILE

Cannot close file  $\langle value \rangle$ .

#### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

The accuracy is specified by **tol**.

## 8 Further Comments

The other models described in Section 3 can be fitted using the generalized linear modelling routines `nag_glm_binomial` (g02gbc) and `nag_glm_poisson` (g02gcc).

The case with one case per stratum can be analysed by having a dummy response variable  $y$  such that  $y = 1$  for a case and  $y = 0$  for a control, and fitting a Poisson generalized linear model with a log link and including a factor with a level for each strata. These models can be fitted by using `nag_glm_poisson` (g02gcc).

`nag_condl_logistic` (g11cac) uses mean centering, which involves subtracting the means from the covariables prior to computation of any statistics. This helps to minimize the effect of outlying observations and accelerates convergence. In order to reduce the risk of the sums computed by Howard's algorithm becoming too large, the scaling factor described in Krailo and Pike (1984) is used.

If the initial estimates are poor then there may be a problem with overflow in calculating  $\exp(\beta^T z_i)$  or there may be non-convergence. Reasonable estimates can often be obtained by fitting an unconditional model.

## 9 Example

The data was used for illustrative purposes by Smith *et al.* (1981) and consists of two strata and two covariates. The data is input, the model is fitted and the results are printed.

### 9.1 Program Text

```

/* nag_condl_logistic (g11cac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg11.h>

int main(void)
{
    /* Scalars */
    double dev, tol;
    Integer iprint, exit_status, i, p, j, m, maxit, n, ns, pdz;
    NagError fail;
    Nag_OrderType order;

    /* Arrays */
    double *b = 0, *cov = 0, *sc = 0, *se = 0, *z = 0;
    Integer *ic = 0, *isi = 0, *isz = 0, *nca = 0, *nct = 0;

#ifdef NAG_COLUMN_MAJOR
#define Z(I,J) z[(J-1)*pdz + I - 1]
    order = Nag_ColMajor;
#else
#define Z(I,J) z[(I-1)*pdz + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);
    exit_status = 0;

    Vprintf("g11cac Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[^\\n] ");

    Vscanf("%ld%ld%ld%ld%*[^\\n] ", &n, &m, &ns, &maxit);

```

```

pdz = n;
iprint = 0;
tol = 1e-5;

/* Allocate arrays z, ic, isi and isz */
if ( !(z = NAG_ALLOC(pdz * m, double)) ||
      !(ic = NAG_ALLOC(n, Integer)) ||
      !(isi = NAG_ALLOC(n, Integer)) ||
      !(isz = NAG_ALLOC(m, Integer)) )
{
  Vprintf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

if (order == Nag_ColMajor)
  pdz = n;
else
  pdz = m;

for (i = 1; i <= n; ++i)
{
  Vscanf("%ld%ld", &isi[i-1], &ic[i-1]);
  for (j = 1; j <= m; ++j)
    Vscanf("%lf", &Z(i,j));
  Vscanf("%*[\n] ");
}

for (j = 1; j <= m; ++j)
  Vscanf("%ld", &isz[j-1]);

Vscanf("%ld%*[\n] ", &p);

/* Allocate other arrays */
if ( !(b = NAG_ALLOC(p, double)) ||
      !(cov = NAG_ALLOC(p*(p+1)/2, double)) ||
      !(sc = NAG_ALLOC(p, double)) ||
      !(se = NAG_ALLOC(p, double)) ||
      !(nca = NAG_ALLOC(ns, Integer)) ||
      !(nct = NAG_ALLOC(ns, Integer)) )
{
  Vprintf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

for (j = 1; j <= m; ++j)
  Vscanf("%lf", &b[j-1]);
Vscanf("%*[\n] ");

g11cac(order, n, m, ns, z, pdz, isz, p, ic, isi, &dev, b, se, sc,
        cov, nca, nct, tol, maxit, iprint, 0, &fail);
if (fail.code != NE_NOERROR)
{
  Vprintf("Error from g11cac.\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}

Vprintf("\n");
Vprintf(" Deviance = %13.4e\n", dev);
Vprintf("\n");

Vprintf(" Strata      No. Cases   No. Controls\n");
Vprintf("\n");
for (i = 1; i <= ns; ++i)
  Vprintf("      %3ld          %2ld          %2ld          \n",
          i, nca[i-1], nct[i-1]);
Vprintf("\n");

```

```

Vprintf(" Parameter      Estimate      Standard Error\n");
Vprintf("\n");
for (i = 1; i <= p; ++i)
  Vprintf("%5ld          %8.4f          %8.4f          \n", i, b[i-1], se[i-1]);
END:
if (b) NAG_FREE(b);
if (cov) NAG_FREE(cov);
if (sc) NAG_FREE(sc);
if (se) NAG_FREE(se);
if (z) NAG_FREE(z);
if (ic) NAG_FREE(ic);
if (isi) NAG_FREE(isi);
if (isz) NAG_FREE(isz);
if (nca) NAG_FREE(nca);
if (nct) NAG_FREE(nct);

return exit_status;
}

```

## 9.2 Program Data

g11cac Example Program Data

```

7 2 2 10
1 0 0 1
1 0 1 2
1 1 0 1
1 1 1 3
2 0 0 1
2 1 1 0
2 1 0 2

1 1 2
0.0 0.0

```

## 9.3 Program Results

g11cac Example Program Results

Deviance = 5.4749e+00

Strata	No. Cases	No. Controls
1	2	2
2	1	2

Parameter	Estimate	Standard Error
1	-0.5223	1.3901
2	-0.2674	0.8473

---